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Soft interaction at high energy and $N = 4$ SYM

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ABSTRACT: In this paper we show that the $N=4$ SYM total cross section violates the Froissart theorem, and in the huge range of energy this cross section is proportional to $s^{1/3}$. The graviton reggeization will change this increase to the normal logarithmic behavior $\sigma \propto \ln^2 s$. However, we demonstrated that this happens at ultra high energy, much higher than the LHC energy. In the region of accessible energy we need to assume that there is a different source for the total cross section, with the value of the cross section about 40 mb. With this assumption we successfully describe σ_{tot} , σ_{el} and σ_{diff} for the accessible range of energy from the fixed target Fermilab to the Tevatron energies. It turns out that the $N=4$ SYM mechanism can be responsible only for a small part of the inelastic cross section for this energy region (about $2mb$). However, at the LHC energy the $N=4$ SYM theory can describe the multiparticle production with $\sigma_{in} \approx 30mb$. The second surprise is the fact that the total cross section and the diffraction cross section can increase considerably from the Tevatron to the LHC energy. The bad description of B_{el} gives the strong argument that the non $N=4$ SYM background should depend on energy. We believe that we have a dilemma: to find a new mechanism for the inelastic production in the framework of $N=4$ SYM other than the reggeized graviton interaction, or to accept that $N=4$ SYM is irrelevant to any experimental data that has been measured before the LHC era.

KEYWORDS: Phenomenology of Field Theories in Higher Dimensions

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1 Introduction

At the moment N=4 SYM is the unique theory which allows us to study theoretically the regime of the strong coupling constant [1]. Therefore, in principle, considering the high energy scattering amplitude in N=4 SYM, we can guess which physics phenomena could be important in QCD, in the limit of the strong coupling. The attractive feature of this theory, is that N=4 SYM with small coupling leads to normal QCD like physics (see refs. [2, 3]) with OPE and linear equations for DIS as well as the BFKL equation for the high energy amplitude. The high energy amplitude reaches the unitarity limit: black disc regime, in which half of the cross section stems from the elastic scattering and half relates to the processes of the multiparticle production.

However, the physical picture in the strong coupling region turns out to be completely different [4–6, 8–10], in the sense that there are no processes of the multiparticle production in this region, and the main contribution stems from elastic and quasi-elastic (diffractive) processes when the target (proton) either remains intact, or is slightly excited. Such a picture not only contradicts the QCD expectations [11–16], but also contradicts available experimental data.

On the other hand, the main success of N=4 SYM has been achieved in the description of the multiparticle system such as the quark-gluon plasma and/or the multiparticle system at fixed temperature [17–20]. Therefore, we face a controversial situation: we know a lot about something that cannot be produced.

The goal of this paper is to evaluate the scale of the disaster, comparing the predictions of the N=4 SYM with the experimental data. We claim that at least half of the total cross section at the Tevatron energy has to stem from a different source than the N=4 SYM.

Before discussing predictions of the N=4 SYM for high energy scattering, we would like to draw the reader's attention that there exists two different regions of energy that we have to consider in N=4 SYM: $(2/\sqrt{\lambda})\alpha's < 1$ and $(2/\sqrt{\lambda})\alpha's > 1$ ($\lambda = 4\pi g_s N_c$ where g_s

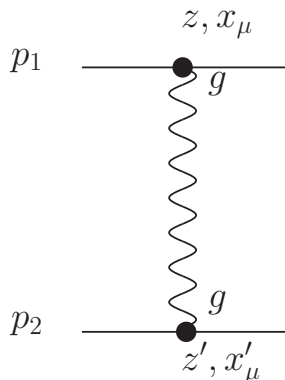


Figure 1. The one graviton (1GE) exchange.

is the string coupling and N_c is the number of colors). In the first region, the multiparticle production has a very small cross section, and it can be neglected. However, in the second region the graviton reggeization leads to the inelastic cross section that is rather large, and at ultra high energies the scattering amplitude reveals all of the typical features of the black disc regime: $\sigma_{el} = \sigma_{tot}/2$ and $\sigma_{in} = \sigma_{tot}/2$.

Therefore, the formulation of the main result of this paper is the following: at the accessible energies the amplitude is in the first region, and at least half of the total cross section at the Tevatron energy has to stem from a different source than the N=4 SYM. However, at the LHC energy the N=4 SYM mechanism can be responsible for about 2/3 of the total cross section and, perhaps, at the LHC the final states will be produced with the typical properties of the N=4 SYM.

2 High energy scattering in $N = 4$ SYM

2.1 Eikonal formula

The main contribution to the scattering amplitude at high energy in N=4 SYM, stems from the exchange of the graviton.¹ The formula for this exchange has been written in ref. [6, 8, 10]. In $AdS_5 = AdS_{d+1}$ space this amplitude has the following form (see figure 1)

$$A_{1GE}(s, b; z, z') = g_s^2 T_{\mu\nu}(p_1, p_2) G_{\mu\nu\mu'\nu'}(u) T_{\mu'\nu'}(p_1, p_2) \xrightarrow{s \gg \mu^2} g_s^2 s^2 z^2 z'^2 G_3(u) \quad (2.1)$$

where $T_{\mu,\nu}$ is the energy-momentum tensor, and G is the propagator of the massless graviton. The last expression in eq. (2.1), reflects the fact that for high energies, $T_{\mu,\nu} = p_{1,\mu} p_{1,\nu}$ and at high energies the momentum transferred $q^2 \rightarrow q_{\perp}^2$ which led to $G_3(u)$ (see refs. [6, 10]). In AdS_5 the metric has the following form

$$ds^2 = \frac{L^2}{z^2} \left(dz^2 + \sum_{i=1}^d dx_i^2 \right) = \frac{L^2}{z^2} (dz^2 + d\vec{x}^2) \quad (2.2)$$

¹Actually, the graviton in this theory is reggeized [5], but it is easy to take this effect into account (see refs. [4, 5, 7]) and eq. (2.9) below.

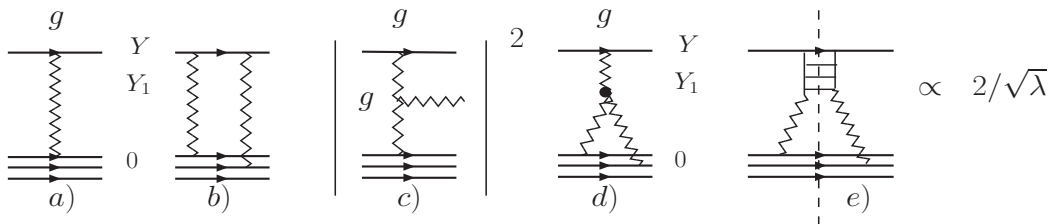


Figure 2. The diagrams for nucleon-nucleon interaction in N=4 SYM. Figure 2-a and figure 2-b show the exchange of one and two gravitons that are included in the eikonal formula of eq. (2.6), while other diagrams give the examples of corrections to the eikonal formula.

and u is a new variable which is equal to

$$u = \frac{(z - z')^2 + (\vec{x} - \vec{x}')^2}{2 z z'} = \frac{(z - z')^2 + b^2}{2 z z'} \quad (2.3)$$

and

$$G_3(u) = \frac{1}{4\pi} \frac{1}{\left\{1 + u + \sqrt{u(u+2)}\right\}^2 \sqrt{u(u+2)}} \quad (2.4)$$

where b is the impact parameter (see figure 1).

As one can see from eq. (2.1) the one graviton exchange amplitude is real. As has been discussed [5] the graviton reggeization leads to a small imaginary part, and the amplitude can be re-written in the form [5, 10]

$$\tilde{A}_{1GE}(s, b; z, z') \equiv \frac{A_{1GE}(s, b; z, z')}{s} = g_s^2 (1 + i\rho) s z z' G_3(u) \quad (2.5)$$

where $\rho = 2/\sqrt{\lambda} \ll 1$. \tilde{A}_{1GE} steeply increases with energy s and has to be unitarized using the eikonal formula [6, 7, 10]

$$A_{\text{eikonal}}(s, b; z, z') = i \left(1 - \exp \left(i \tilde{A}_{1GE}(s, b; \text{eq. (2.5)}) \right) \right) \quad (2.6)$$

In ref. [10] it was argued that AdS/CFT correspondence leads to the corrections to eq. (2.6) which are small ($\propto 2/\sqrt{\lambda}$). The unitarity constraints for eq. (2.6) has the form

$$2 \text{Im} \mathcal{A}_{\text{eikonal}}(s, b; z, z') = |A_{\text{eikonal}}(s, b; z, z')|^2 + \mathcal{O} \left(\frac{2}{\sqrt{\lambda}} \right) \quad (2.7)$$

The eikonal formula of eq. (2.6) as well as the unitarity constraint of eq. (2.7) are illustrated in figure 2. One can see that the diagrams shown in this figure have the following contributions:

$$A(\text{figure 2 - a}) \propto g_s^2 s \approx \frac{s}{N_c^2};$$

$$A(\text{figure 2 - b}) \propto (g_s^2 s)^2 \approx \left(\frac{s}{N_c^2} \right)^2;$$

$$\begin{aligned}
 A(\text{figure 2-c}) &\propto g_s^2 (g_s^2 s)^2 \approx \frac{1}{N_c^2} \left(\frac{s}{N_c^2}\right)^2; \\
 A(\text{figure 2-d and figure 2-e}) &\propto \frac{2}{\sqrt{\lambda}} \left(\frac{s}{N_c^2}\right)^2.
 \end{aligned}
 \tag{2.8}$$

Therefore, the contributions that lead to a violation of the eikonal formula are small, at least as small as $2/\sqrt{\lambda}$. It is interesting to notice that actually they stem from the processes of the diffraction dissociation (see figure 2-e rather than from the processes of the multiparticle productions (see figure 2-c).

Eq. (2.5) provides the simple method to take into account the reggeization of the graviton, in order to understand the main property of the scattering amplitude. However in our description of the experimental data, we will use the exact form of the amplitude for the exchange of the reggeized graviton (see refs. [5, 10]), namely,

$$\tilde{A}_{1GE}(s, b; z, z') = g_s^2 (1 + i\rho) \frac{1}{4\pi} \frac{(z z' s)^{1-\rho}}{\sqrt{u(u+2)}} \sqrt{\frac{\rho}{\pi \ln(s z z')}} \exp\left(-\frac{\ln^2(1 + u + \sqrt{u(u+2)})}{\rho \ln(s z z')}\right)
 \tag{2.9}$$

eq. (2.9) gives the description of one reggeized graviton in the limit $s \rightarrow \infty$ with $\lambda \gg 1$ while the simple formula of eq. (2.5) describes the one graviton exchange for $\lambda \rightarrow \infty$ but $s \gg 1/\alpha'$.

2.2 Nucleon-nucleon high energy amplitude

Discussing the hadron interaction at high energy, we need to specify the correct degrees of freedom that diagonalize the interaction matrix. We assume that a nucleon consists of N_c quarks (N_c colorless dipoles) that interact with each other with the eikonal formula of eq. (2.6), namely,

$$\begin{aligned}
 A_{NN}(s, b) &= \int dz dz' \prod_{i=1}^{N_c} d^2 r_i \prod |\Psi(r_i, z)|^2 \prod_{i=1}^{N_c} d^2 r'_i \prod |\Psi(r_i, z')|^2 \\
 &\quad \times i \left(1 - \exp\left(i N_c^2 \tilde{A}_{1GE}(s, b; z, z' | \text{eq. (2.5)})\right)\right) \\
 &= i \int dz dz' \Phi(z) \Phi(z') \left(1 - \exp\left(i g^2 N_c^2 (1 + i\rho) z^2 z'^2 G_3(u)\right)\right)
 \end{aligned}
 \tag{2.10}$$

where

$$\Phi(z) = \int d^2 r \prod |\Psi(r, z)|^2
 \tag{2.11}$$

and $\rho = 2/\sqrt{\lambda}$.

In eq. (2.10) the only unknown ingredient is $\Psi(r_i, z)$. We can reconstruct this wave function using the Witten formula [21], namely,

$$\Psi(r, z) = \frac{\Gamma(\Delta)}{\pi \Gamma(\Delta - 1)} \int d^2 r' \left(\frac{z}{z^2 + (\vec{r} - \vec{r}')^2}\right)^\Delta \Psi(r') \text{ with } \Delta_\pm = \frac{1}{2} \left(d \pm \sqrt{d^2 + 4m^2}\right)
 \tag{2.12}$$

where $\Psi(r')$ is the wave function of the dipole inside the proton on the boundary. For simplicity and to make all calculations more transparent, we choose $\Psi(r') = K_0(Qr')$.

The value of the parameter Q can be found from the value of the electromagnetic radius of the proton ($Q \approx 0.3 \text{ GeV}^{-1}$).

In this presentation, we follow the formalism of ref. [10], namely using the formulae **3.198**, **6.532(4)**, **6.565(4)** and **6.566(2)** from the Gradstein and Ryzhik tables, ref. [22]. Introducing the Feynman parameter (t), we can rewrite eq. (2.12) in the form

$$\begin{aligned}
 \Psi(r, z) &= \frac{\Gamma(\Delta)}{\pi\Gamma(\Delta-1)} \int \xi d\xi d^2 r' \frac{J_0(Q\xi)}{\xi^2 + r'^2} \left(\frac{z}{z^2 + (\vec{r}' - \vec{r}')^2} \right)^\Delta \\
 &= \frac{\Gamma(\Delta+1)}{\pi\Gamma(\Delta-1)} \frac{1}{B(1, \Delta)} \\
 &\quad \times \int \xi d\xi d^2 r' \int_0^1 \frac{dt}{z} t^{\Delta-1} (1-t) J_0(Q\xi) \left(\frac{z}{tz^2 + t(\vec{r}' - \vec{r}')^2 + (1-t)r'^2 + (1-t)\xi^2} \right)^{\Delta+1} \\
 &= \frac{\Gamma(\Delta+1)}{\pi\Delta\Gamma(\Delta-1)} z^\Delta \int \tilde{\xi} d\tilde{\xi} \int_0^1 dt \frac{1}{(1-t)^\Delta} J_0 \left(Q\sqrt{\frac{t}{1-t}} \tilde{\xi} \right) \left(\frac{1}{r^2 + \kappa(t, z, \tilde{\xi})} \right)^\Delta \quad (2.13)
 \end{aligned}$$

with $\kappa(t, z, \xi) = (tz^2 + \xi^2)/(1-t)$ and $\tilde{\xi} = \xi(\sqrt{1-t}/\sqrt{t})$.

The amplitude $\tilde{A}_{1GE}(s, b; z, z' \text{ eq. (2.5)})$ depends only on z and z' , and we need to find $\int |\Psi(r, z)|^2 d^2 r$. From eq. (2.13), one can see that we have to evaluate the integral

$$\begin{aligned}
 \pi \int dr^2 \left(\frac{1}{r^2 + \kappa(t, z, \tilde{\xi})} \right)^\Delta \left(\frac{1}{r^2 + \kappa(t', z, \tilde{\xi}')} \right)^\Delta &= \\
 &= \pi B(1, 2\Delta-1) {}_2F_1 \left(1, \Delta, 2\Delta-1, 1 - \frac{\kappa(t, z, \tilde{\xi})}{\kappa(t', z, \tilde{\xi}')} \right) \\
 &\approx \pi \frac{1}{2\Delta-1} \frac{\kappa(t, z, \tilde{\xi})}{\left(\kappa(t, z, \tilde{\xi}) \kappa(t', z, \tilde{\xi}') \right)^\Delta} \quad (2.14)
 \end{aligned}$$

where we used **3.197** of ref. [22].

In the last equation we assumed that $\kappa(t, z, \xi)/\kappa(t', z, \xi')$ is close to unity, since the integral has a symmetry with respect to $\xi \rightarrow xi'$, and $t \rightarrow t'$. The simplified form allows us to reduce the integral for $\Phi(z)$ (see eq. (2.11)), to the form

$$\begin{aligned}
 \Phi(z) &= z^{2\Delta} \left(\frac{\Gamma(\Delta+1)}{\pi\Gamma(\Delta-1)} \right)^2 \frac{\pi}{2\Delta-1} \int \tilde{\xi} d\tilde{\xi} J_0 \left(Q\sqrt{\frac{t}{1-t}} \tilde{\xi} \right) dt \frac{1}{(tz^2 + \tilde{\xi}^2)^\Delta} \\
 &\quad \times \int \tilde{\xi}' d\tilde{\xi}' J_0 \left(Q\sqrt{\frac{t'}{1-t'}} \tilde{\xi}' \right) dt' (1-t') \frac{1}{(t'z^2 + (1-t)\tilde{\xi}'^2)^{\Delta-1}} \\
 &= \left(\frac{2\alpha'}{\rho} \right)^{\Delta-2} \frac{(\Delta-1)^2}{\Gamma(\Delta)\Gamma(\Delta-1)\pi} \frac{2Q^2 2^{3-2\Delta} z^3}{2\Delta-1} \int_0^1 dt \left(\sqrt{\frac{t}{1-t}} Q \right)^{\Delta-1} K_{\Delta-1} \left(\sqrt{\frac{t}{1-t}} Qz \right) \\
 &\quad \times \int_0^1 dt' \left(\sqrt{\frac{t'}{1-t'}} Q \right)^{\Delta-2} K_{\Delta-2} \left(\sqrt{\frac{t'}{1-t'}} Qz \right) \quad (2.15)
 \end{aligned}$$

In the last equation we included the factor $\left(\frac{2\alpha'}{\rho}\right)^{2\Delta-4}$, which recovers the correct dimension of the wave function. The origin of this factor is simple: we assumed for simplicity in all our previous calculations, that $L = 1$ in AdS_5 . Since $L^2 = \alpha'\sqrt{\lambda} = \alpha'2/\rho$, this factor is the way to take into account the fact that $L^2 \neq 1$.

2.3 Qualitative features of high energy scattering

From eq. (2.10) one can see that $A_{NN}(s, b)$ tends to 1 in the region of b from $b = 0$ to $b = b_0(s)$. Since $G_2(u) \rightarrow 1/b^6$ at large b , we can conclude that $A_{NN}(s, b) \propto s/b^6$ at $b \gg z^2$. Therefore, $b_0^2 \propto s^{1/3}$ and the nucleon-nucleon scattering amplitude generates the total cross section

$$\sigma_{\text{tot}} = 2 \int d^2b \text{Im}A_{NN}(s, b) \propto s^{1/3} \tag{2.16}$$

in obvious violation of the Froissart theorem [23]. As has been shown in refs. [5, 10] the Froissart theorem can be restored if we consider the string theory which leads to N=4 SYM in the limit of the weak graviton interaction. In this string theory the graviton with positive t (t is the momentum transferred along the graviton in figure 1) lies on the Regge trajectory with the intercept $\alpha'/2$ which corresponds to the closed string. On the other hand in AdS_5 the Einstein equation has the form

$$R_{\mu,\nu} - \frac{1}{2}R g_{\mu,\nu} = \frac{6}{L^2} g_{\mu,\nu} = \frac{6}{\sqrt{\lambda}\alpha'} g_{\mu,\nu} \tag{2.17}$$

where $R_{\mu,\nu}$ is the Ricci curvature tensor and R is the Ricci curvature. In consequence of eq. (2.17) the graviton has the mass [24] $m_{\text{graviton}}^2 = 4/\sqrt{\lambda}\alpha' = 2\rho/\alpha'$ and the intercept $2 - m_{\text{graviton}}^2(\alpha'/2) = 2 - 2/\sqrt{\lambda} = 2 - \rho$.

The fact that the graviton has mass results in the different behavior of the gluon propagator at large b , namely, at large b it shows the exponential decrease $G(b) \rightarrow \exp(-m_{\text{graviton}} b) = \exp(-\sqrt{2\rho}b^2)$. Such behavior restores the logarithmic dependence of the cross section at high energy, in the agreement with the Froissart theorem but nevertheless we expect a wide range of energies where the cross section behaves as $s^{1/3}$. Experimentally, the total cross section in the energy range from fixed target experiment at FNAL to the Tevatron energy, has $\sigma_{\text{tot}} \propto s^{0.1}$. Therefore, we expect that the cross section cannot be described by eq. (2.10).

We replace $G_3(u)$ in eq. (2.10) and in eq. (2.9) by

$$\tilde{A}_{1GE}(s, b; z, z') \longrightarrow \tilde{A}_{1GE}(s, b; z, z') e^{-\sqrt{2\rho/\alpha'} b} \tag{2.18}$$

to take into account the effect of the graviton reggeization. Introducing this equation we are able to specify the kinematic energy range, where we expect the $s^{1/3}$ behavior of the total cross section.

3 The comparison with the experimental data

As we have discussed, we face two main difficulties in our attempts to describe the experimental data in N=4 SYM: the small value of the cross section of the multiparticle

production and the violation of the Froissart theorem. The scale of both phenomena is given by the value of $2/\sqrt{\lambda}$ (see eq. (2.7) and eq. (2.18)) and, if this parameter is not small, we, perhaps, have no difficulties at all. On the other hand, the N=4 SYM could provide the educated guide only for $2/\sqrt{\lambda} \ll 1$ since it has an analytical solution for such λ . Therefore, the goal of our approach is to describe the experimental data assuming that $2/\sqrt{\lambda}$ is reasonably small (say $2/\sqrt{\lambda} \leq 0.3$), and to evaluate the scale of the cross section for the multiparticle production. As has been mentioned, the multiparticle production can be discussed in N=4 SYM since the confinement of the quarks and gluon, we believe, is not essential for these processes.

We use eq. (2.7) with eq. (2.15) to calculate the physical observables, namely,

$$\sigma_{\text{tot}} = \sigma_0 + 2 \int d^2b \text{Im} A(s, b) \quad (3.1)$$

$$= \sigma_0 + \frac{4}{\rho\alpha'} \int d^2b \int \Phi(z) \Phi(z') dz dz' \text{Re} \left\{ 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | \text{eq. (2.9)}) \right) \right\}$$

$$\sigma_{el} = \int d^2b |A_0(b) + A(s, b)|^2 \quad (3.2)$$

$$= \int d^2b |A_0(b) + \int \Phi(z) \Phi(z') dz dz' i \left\{ 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | \text{eq. (2.9)}) \right) \right\}|^2;$$

$$B_{el} = \frac{\int d^2b b^2 |A_0(b) + \int \Phi(z) \Phi(z') dz dz' i \left\{ 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | \text{eq. (2.9)}) \right) \right\}|^2}{\int d^2b |A_0(b) + \int \Phi(z) \Phi(z') dz dz' i \left\{ 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | \text{eq. (2.9)}) \right) \right\}|^2}; \quad (3.3)$$

As has been expected, it turns out that in the experimental accessible region of energies, the cross section given in eq. (2.10) shows the $s^{1/3}$ behavior for a wide range of parameters: $g^2 = 0.01 \div 1$, $Q = 0.2 \div 1 \text{ GeV}^{-1}$ and $\rho = 0 \div 0.3$. Our choice of the parameters reflects the theoretical requirements for N=4 SYM, where we can trust this approach, namely, $g_s \ll 1$ while $g_s N_c > 1$. The values of σ_{tot} from eq. (2.10) with \tilde{A} from eq. (2.9) are small for $W = \sqrt{s} = 20 \text{ GeV}$ but it increases and becomes about $20 - 30 \text{ mb}$ at the Tevatron energy. Facing the clear indication that we need an extra contribution to the total cross section in eqs. (3.1)–(3.3), we introduce the contribution of the non N=4 SYM origin (σ_0 and the amplitude $A_0(b)$).

It should be mentioned that we have also a hidden parameter Δ in the wave function of the proton. At the moment theoretically we know only that $\Delta > 2$. This constraint stems from the convergence of the integral for the norm of the proton wave function (see refs. [4, 9, 21, 25]). We have tried several values of Δ and $\Delta = 3$ is our best choice (see figure 8).

For a purely phenomenological background $A_0(b)$ we wrote the simplest expression

$$A_0(b) = i \frac{\sigma_0}{4\pi B_0} \exp(-b^2/2B_0) \quad (3.4)$$

where B_0 is the slope for the elastic cross section.

With these two new parameters σ_0 and B_0 , we tried to describe the data. The results are shown in figure 3, figure 4 and figure 5.

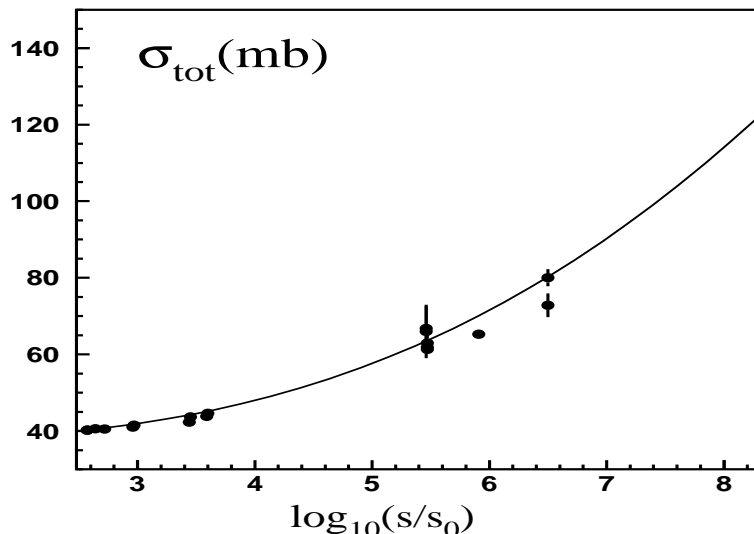


Figure 3. The description of the total cross section $\sigma_{\text{tot}} = (\sigma_{\text{tot}}(pp) + \sigma_{\text{tot}}(p\bar{p}))/2$ with $Q = 0.35 \text{ GeV}$, $g = g_s^2 N_c^2 = 0.1$, $\rho = 0.25$, $\Delta = 3$ and with $\sigma_0 = 37.3 \text{ mb}$.

From these pictures one can see that for the total and elastic cross section, we obtain a good agreement with the experimental data, whereas for the elastic slope (B_{el}), the description is in contradiction with the experimental data. First we would like to understand the main ingredients of the total cross section. For doing so we need to estimate the cross section of the diffractive dissociation. In the N=4 SYM approach;

$$\sigma_{\text{diff}} = \frac{2}{\rho\alpha'} \int d^2b \int \Phi(z) \Phi(z') dz dz' \left| 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | eq. (2.9)) \right) \right|^2 - \left| \frac{2}{\rho\alpha'} \int d^2b \int \Phi(z) \Phi(z') dz dz' \left| 1 - \exp \left(iN_c^2 \tilde{A}_{1GE}(s, b, z, z' | eq. (2.9)) \right) \right| \right|^2 \quad (3.5)$$

In eq. (3.5) $\sigma_{\text{diff}} = \sigma_{sd} + \sigma_{dd}$ where σ_{sd} and σ_{dd} are cross sections of single and double diffraction respectively. Our predictions for σ_{diff} have been plotted in figure 6, where curve 1 is the result of the calculation using eq. (3.5), and curve 2 is the same except for the addition of 4 mb from the diffractive cross section, which is of non N=4 SYM origin. In table 1, we compare our predictions with the phenomenological models that do not take into account the N=4 SYM physics. The result of this comparison is interesting, since our simple estimates show that the cross section of the diffractive production, could considerably grow from the Tevatron to the LHC energy. We want to recall that the unitarity constraints of eq. (2.7), lead to $|A(s, b; z, z')| \leq 2$ and $\sigma_{\text{tot}} = \sigma_{el}$.

As far as the inelastic cross section is concerned, one can see that the inelastic cross section of the N=4 SYM origin $\sigma(\text{N=4 SYM}) = \sigma_{\text{tot}} - \sigma_{el} - \sigma_{\text{diff}} - \sigma_{0,in}$ is about 2 mb both for RHIC and the Tevatron energy, and grows to 30 mb at the LHC energy. Therefore, we can observe some typical features of the N=4 SYM theory, which only start at the LHC energy.

The above estimates are based on the background that does not depend on energy. However, figure 4 illustrates that the non N=4 SYM background should also depend on energy. In figure 4 (the upper curve) we plot the elastic slope for the background of eq. (3.4)

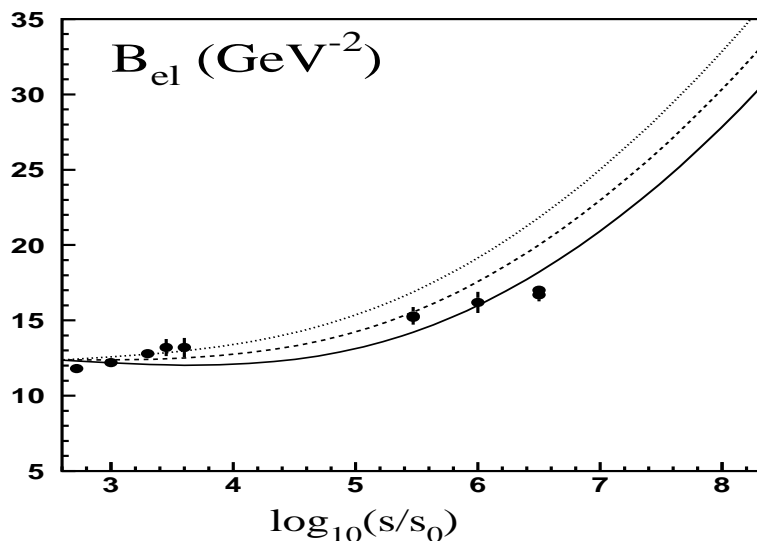


Figure 4. The description of the energy behavior of the elastic slope with the same set of parameters as in figure 3 and with $B_0 = 12.37 \text{ GeV}^{-2}$ (solid curve) and $B_0 = 12.37 + 2\alpha'_P \ln(s/s_0)$ ($\alpha'_P = 0.1 \text{ GeV}^{-2}$ (dashed curve) and $\alpha'_P = 0.2 \text{ GeV}^{-2}$ (dotted curve)).

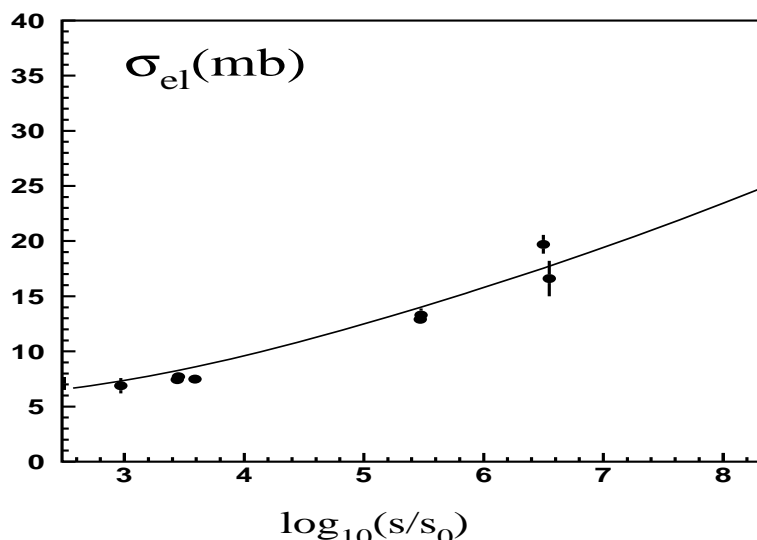


Figure 5. The description of the energy behavior of the elastic cross section with the same set of parameters as in figure 3 and $B_0 = 12.37 \text{ GeV}^{-2}$.

but with $B_0 = 12.37 + 2\alpha'_P \ln(s/s_0)$. This amplitude corresponds to the exchange of the Pomeron with intercept 1 which generates the constant cross section but leads to a shrinkage of the diffraction peak. One can see that we are able to describe the slope in such a model.

In figure 7 and figure 8 we plot the dependence of σ_{tot} and σ_{el} on the parameters of our approach to illustrate the sensitivity of our descriptions of the experimental data to their values.

The results of our calculation show that in the large range of energies, the N=4 SYM scattering amplitude behaves as $s^{1/3}$ with a rather small coefficient in front. The graviton

	Tevatron			LHC		
	GLMM	KMR	LP	GLMM	KMR	LP
$\sigma_{\text{tot}}(\text{mb})$	73.29	74.0	83.2	92.1	88.0	124.9
$\sigma_{el}(\text{mb})$	16.3	16.3	17.5	20.9	20.1	24.4
$\sigma_{sd} + \sigma_{dd}(\text{mb})$	15.2	18.1	24.4	17.88	26.7	42.3
$(\sigma_{el} + \sigma_{sd} + \sigma_{dd}) / \sigma_{\text{tot}}$	0.428	0.464	0.504	0.421	0.531	0.536

Table 1. Comparison of the GLMM ([26]) and KMR [27] models and our estimates (LP).

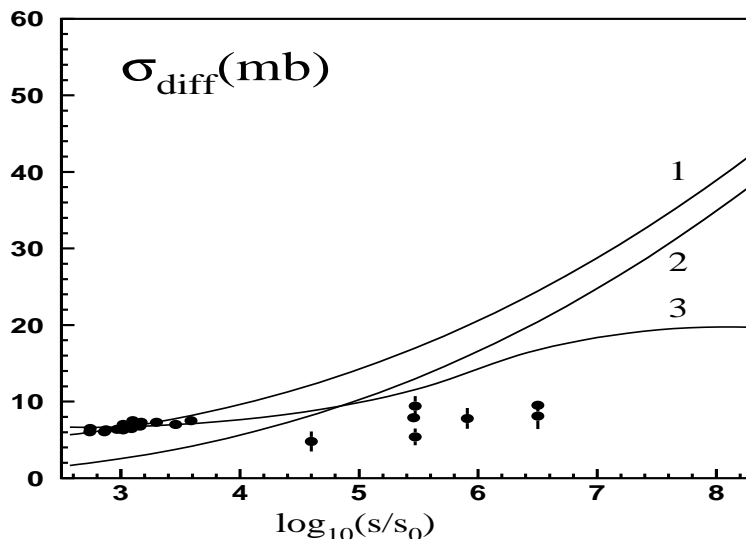


Figure 6. The description of the energy behavior of the diffraction production cross section $\sigma_{\text{diff}} = \sigma_{sd} + \sigma_{dd}$ with the same set of parameters as in figure 5. σ_{sd} and σ_{dd} are cross sections of single and double diffraction production respectively. The curve 2 shows the N=4 SYM contribution to the diffraction production while the curve 1 corresponds to the N=4 SYM prediction plus 4 mb for the cross section of a different source than N=4 SYM. The data are only for single diffraction production. In curve 3 we plot the estimates of ref. [26] for σ_{diff} .

reggeization that will stop the anti-Froissart behavior at ultra high energies, does not show up at the accessible range of energy from the fixed target Fermilab energy, until the Tevatron energy. This reggeization can be measured, perhaps, only at the LHC energy.

4 Conclusion

In this paper we show that the N=4 SYM total cross section violates the Froissart theorem, and in the huge range of energy this cross section is proportional to $s^{1/3}$. The graviton reggeization will change this increase to the normal logarithmic behavior $\sigma \propto \ln^2 s$. However, we demonstrated that this happens at ultra high energy, much higher than the LHC energy for reasonably low $2/\sqrt{\lambda} \approx 0.25$.

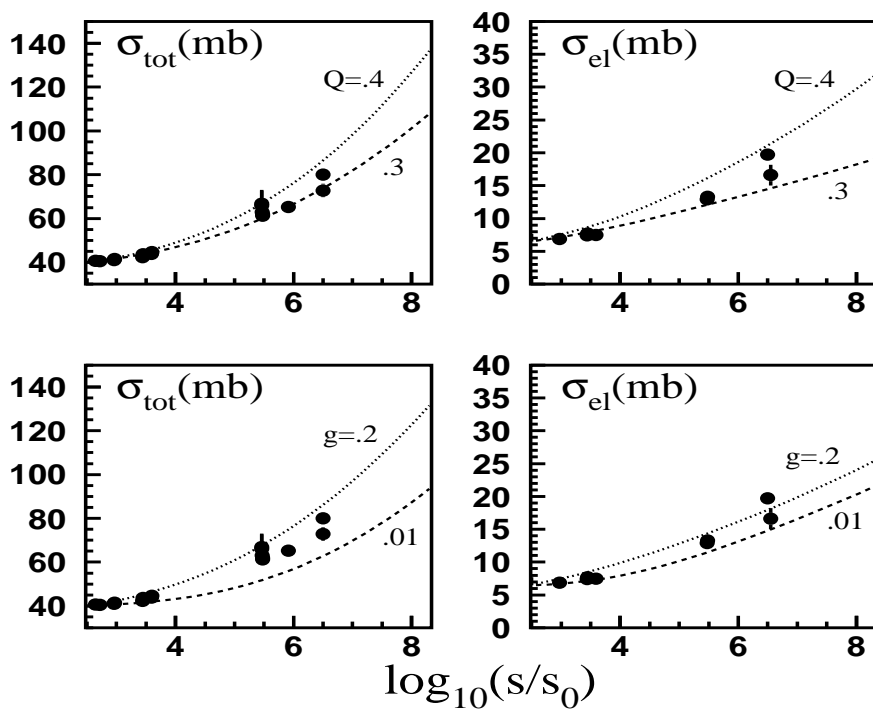


Figure 7. The dependence of the description on Q and $g = N_c^2 g^2$.

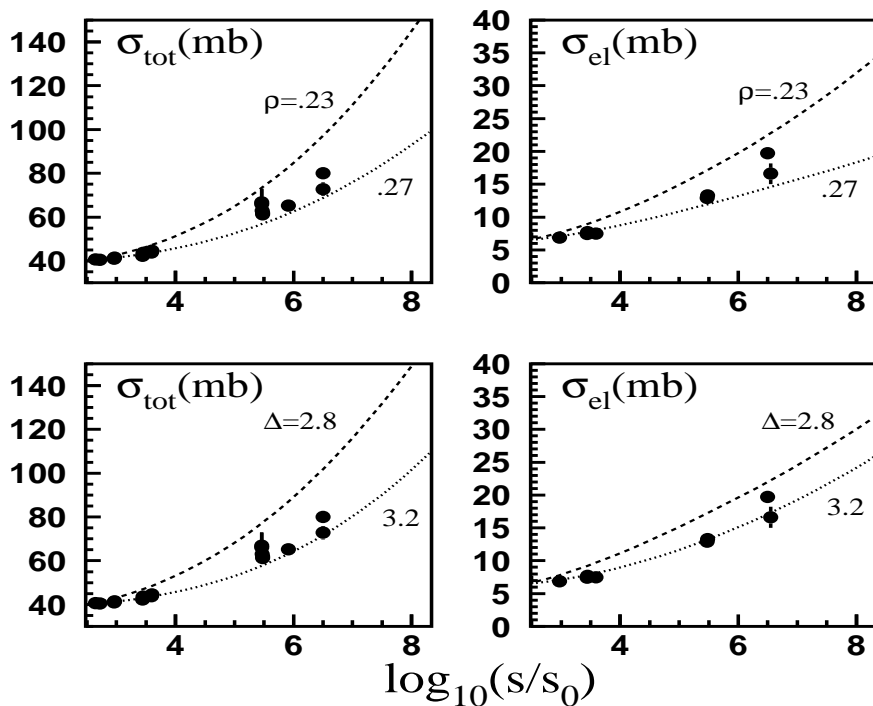


Figure 8. The dependence of the description on ρ and Δ .

We need to assume that there is a different source for the total cross section, with the value of the cross section about 40 mb. With this assumption we successfully describe σ_{tot} , σ_{el} and σ_{diff} for the accessible range of energy from the fixed target Fermilab to the Tevatron energies. The N=4 SYM mechanism is responsible only for a small part of the inelastic cross section for this energy region (about $2mb$). However, at the LHC energy the N=4 SYM theory can lead to a valuable contribution to the inelastic cross section, namely, $\sigma_{in} \approx 30mb$ which is about a quarter of the total inelastic cross section. The second surprise is the fact that the total cross section and the diffraction cross section can increase considerably from the Tevatron to the LHC energy. The bad description of B_{el} gives the strong argument that the non N=4 SYM background should depend on energy.

It means that at RHIC energies, the N=4 SYM part of the inelastic cross section is negligible and the quark-gluon plasma is created by the mechanism outside of N=4 SYM. For the LHC energy, we can expect that N=4 SYM is responsible for the inelastic cross section of about $\sigma_{in}(N = 4 \text{ SYM}) = 30 \text{ mb}$ out of $\sigma_{\text{tot}} = 121.9 \text{ mb}$.

We believe that we have a dilemma: to find a new mechanism for the inelastic production in the framework of N=4 SYM other than the reggeized graviton interaction, or to accept that N=4 SYM is irrelevant to description of any experimental data that have been measured before the LHC era, with a chance that even at the LHC it will be responsible only for a quarter (or less) of the total cross section. Deeply in our hearts, we believe in the first way out, and we hope that this paper will draw attention to this challenging problem: searching for a new mechanism for multiparticle production in N=4 SYM.

We wish to draw your attention to the fact that the scattering amplitude can change considerably from the Tevatron to LHC energy (see table 1). Therefore, all claims that we can give reliable predictions for the values of the cross sections at the LHC energy and even of the survival probability for the diffractive Higgs production [27] looks exclusively naive and reflects our prejudice rather than our understanding.

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